MATH 147 QUIZ 4 SOLUTIONS

1. Find and classify the critical points for $f(x, y) = x^2 + y^2 - 6x - 14y + 100$. (5 Points)

   We take the partial derivatives with respect to x and y. We have $f_x = 2x - 6$ and $f_y = 2y - 17$,giving us a critical point of $(3, 7)$. We now perform the second derivative test. Seeing that $f_{xx} = 2 = f_{yy}$ and $f_{xy} = 0$, we get $D = 4$, so we have a relative minimum at $(3, 7)$.

2. Find the absolute maximum and minimum values for $f(x, y) = \frac{-2y}{x^2+y^2+1}$ on $D = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 4\}$. (5 points)

   We begin by finding the critical points within the boundary. The partial derivatives are

   $$f_x = \frac{4xy}{(x^2 + y^2 + 1)^2} \text{ and } f_y = \frac{-2(x^2 - y^2 + 1)}{(x^2 + y^2 + 1)^2}$$

   To find out when these give critical points, note that the denominators will never be zero. Thus, we need to only find the zeros of the above functions. In particular, $f_x = 0$ only along the x and y axes. Note that if $y = 0$, then the numerator of f_y is $-2(x^2 + 1)$, which cannot be zero. Thus, we restrict our attention to the case that $x = 0$, where now the numerator of f_y is $-2(1 - y^2)$ which has zeros at $y^2 = 1$, meaning our critical points are $(0, 1)$ and $(0, -1)$. As for the boundary, note that along $x^2 + y^2 = 4$, $f(x, y)$ is of the form $f(x, y) = -2y/5$. This is a linear function in y, with constant derivative, so the absolute extrema will lie on the boundary, that is, $y = \pm 2$. Thus, our 4 points to check are $(0, 1), (0, -1), (0, 2), (0, -2)$. We see that the first two points, producing -1 and 1 respectively, are the absolute extrema, as the points on the boundary only give $-4/5$ and $4/5$ respectively.